6.2 Discrete-Time H_2 Optimization

We illustrate potential issues that can arise with "optimal" but purely discrete-time designs for sampled-data systems. The following example is taken from [CF95, chapter 6].

Example 6.2.1. Consider the sampled-data tracking setup of Fig. 6.1, with P a stable, SISO, second-order system with transfer function

$$P(s) = \frac{1}{(10s+1)(25s+1)}, \tag{6.6}$$

and the reference input r is the unit step. The goal is to minimize the 2-norm (energy) of the CT error signal $e = r - y$,

$$\|e\|_2 = \int_0^\infty |e(t)|^2 dt,$$

so that the plant output tracks the r optimally in this sense. The sampling period is assumed to be $h = 1s$, which is much smaller than the time constants of the plant (10 s and 25 s). The bandwidth of P is less than 0.04 rad/s for a drop of $3dB$ and less than 0.057 rad/s for a drop of 6 dB, see Fig. 6.2, and we are sampling at $2\pi \approx 6.28$ rad/s, which would seem a priori sufficient.

By linearity of the sampling operation, we can pass the S block on the other side of the summing junction, resulting in the discrete-time system shown on the figure as well, with $P_d = SPH$ the step-invariant transformation of P, $\rho = Sr$ and $\epsilon = Se$. Note that after this transformation, the continuous-time signal of interest is not available any more, as it becomes an internal signal in P_d. The discrete-time design approach consists in designing K_d to minimize the 2-norm sampled version ϵ of this signal

$$\|\epsilon\|_2 = \sum_{k=0}^{\infty} |\epsilon[k]|^2.$$

As noted before, this does not provide a guarantee that $\|e\|_2$ will be small.

The discretized plant P_d has the transfer function (recall that $\lambda = 1/z$)

$$P_d(\lambda) = \frac{2.0960 \times 10^{-3} \lambda(\lambda + 1.0478)}{(\lambda - 1.0408)(\lambda - 1.1052)},$$

and it turns out that the optimal discrete-time controller (obtained via discrete-time H_2 optimization) has transfer function

$$K_d(\lambda) = \frac{477.1019(\lambda - 1.1052)(\lambda - 1.0408)}{(\lambda + 1.0478)(\lambda - 1)}.$$

In other words, it cancels the stable poles and zeros of P_d, and adds a pole at $\lambda = 1$ required for step tracking. For this controller, ϵ is the unit impulse

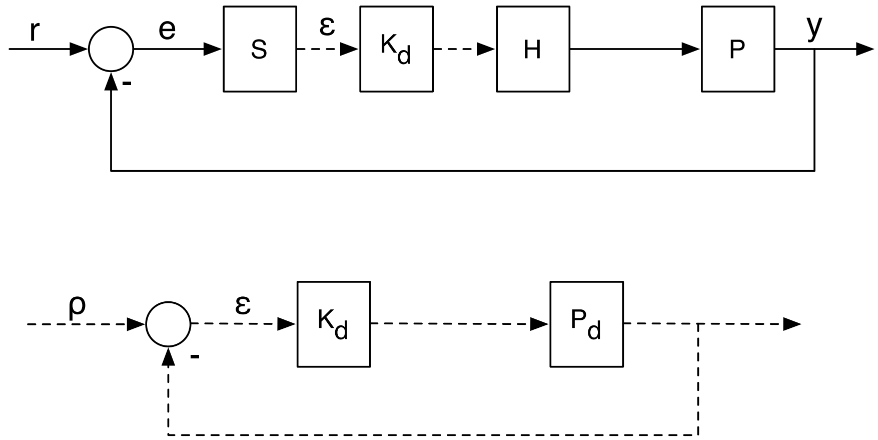

Figure 6.1: Sampled-data tracking system and its discrete-time version.

$\epsilon = \delta_d$, i.e., the discrete plant output requires only one discrete-time step (1 s in real time) to reach its final value. The performance appears therefore to be quite good. Simulating the analog response, which is shown on Fig. 6.3, tells us that the discrete-time analysis is misleading however. It confirms that the signal $y(t)$ is equal to $r(t)$ at the sampling times, but at the expense of large intersample oscillations. The discrete time design over-emphasizes the importance of the sampling instants.

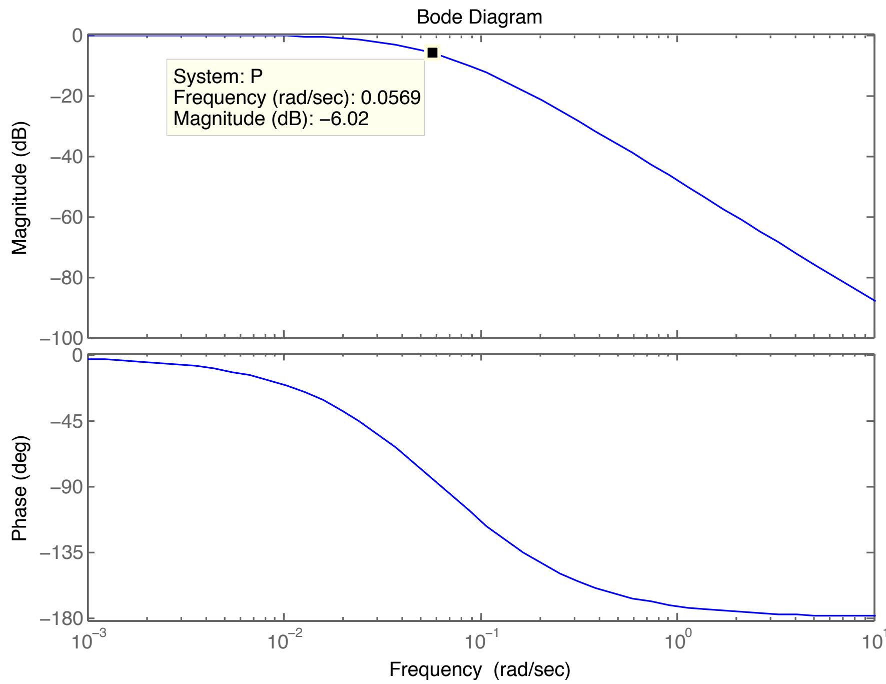

Figure 6.2: Bode plot of the plant 6.6.

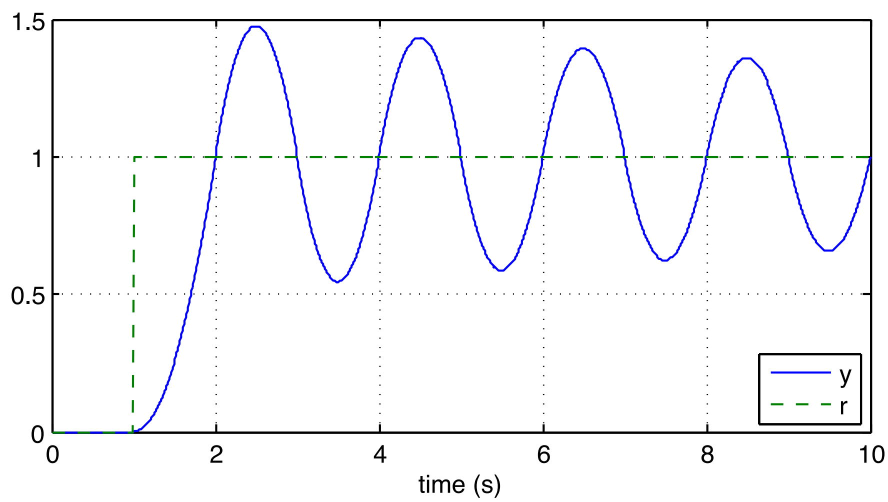

Figure 6.3: Step-response of example 6.2.1.