Planar 3 Degrees of freedom Parallel Manipulator with an Articulated Platform Featuring a Planetary Gearbox

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Abstract—In order to develop new architectures of parallel manipulators, this paper presents the analysis of a new 3 degrees of freedom planar manipulator design. The innovative feature of the presented architecture is the integration of a planetary gearbox with an articulated platform. The addition of this gearbox reduces the number of links controlling the position and the orientation control of the end-effector to two. This manipulator can be described as a ten-bar mechanism with three active revolute joints, 5 passive revolute joints and a passive planetary gearbox joint. First, the architecture design of the manipulator will be described in detail. Then, the direct kinematic analysis will be presented. Finally, the workspace analysis will be performed.

Keywords: Articulated platform, Planetary gearbox, Planar parallel manipulator, Ten-bar mechanism

I. Introduction

In the last few decades, several parallel kinematic designs have been studied for their interesting properties. Most of the previous architectures were design where all the limbs are identical, as presented in papers [1, 2]. The proposed mechanism presents an original design, which combines three actuated limbs in an articulated platform, where only two passive links drive the end-effector, which still has three degrees of freedom. Very few parallel manipulator architectures present an articulated platform, as presented in papers [3, 4].

II. The Kinematic Design

The design presented in this paper is a ten-bar linkage mechanism, as shown in Fig. 1. The legs of the mechanism are not identical and can be described as a three kinematic chain, 2-RRR and 1-RR, connected to a four-bar linkage (C2-C3-C4-C5). This four-bar linkage controls the position of the end-effector. With this architecture, the orientation of the end-effector cannot be controlled by revolute joints only. However the utilization of a planetary gearbox at the C4, C5 and EE junction, instead of a revolute joint, can solve this problem and controls the orientation of the end-effector. The diagram of this mechanism is shown in Fig. 2.

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The first limb is actuated by the revolute joint $R_1$ (angle $\theta_1$), which is connected to the base BB and the body A1. The position of this joint is positioned at the coordinate $(0, 0, 0)$. The link A1 is then connected to the link C2 and C3 by a passive revolute joint $\theta_1$. These two links are part of the four-bar linkage, which controls the position and orientation of the end-effector.

The second limb is actuated by the revolute joint $R_2$ (angle $\theta_2$), which controls the orientation of the link A2 and is placed arbitrarily at the coordinate $(-10, 0, 0)$. The link A2 is then connected to the link B2 by a passive revolute joint $R_7$, which is then connected to the links C2 and C5 by a passive revolute joint $R_7$. 

Figure 1 - Planar 3-DOF with articulated PLATFORM

Figure 2 - Kinematic chain
The third limb is symmetrical with respect to the second limb and is actuated by the revolute joint \( R_3 \) (angle \( \theta_3 \)), placed arbitrarily at the coordinate (10, 0, 0), which connects the link A3 to the base BB. The link A3 is then connected to the limb B3 by a passive revolute joint \( R_6 \). This link is then connected to the links C3 and C4 by a passive revolute joint \( R_8 \).

To simplify the equations, the four-bar mechanism is a parallelogram, where C2 and C4 are chosen to be the same length and similarly C3 and C5 are the same length.

In order to achieve the control of the orientation of the end-effector, a passive planetary gearbox \( G_9 \) connects the end effector to C4 and C5. Moreover, the gear ratio can be adjusted to satisfy specific needs in terms of end-effector to C4 and C5. Moreover, the gear ratio can be adjusted to satisfy specific needs in terms of end-effector orientation.

As shown in Fig. 3, the vectors from the origin to the actuated joints are called \( o_i \) where “i” is the limb number. The vector from the origin to the first set of passive joint is called \( u_i \). The vectors from the origin to the passive joints \( R_7 \) and \( R_8 \) are called \( v_i \) where “i” is associated to the 2nd or 3rd limb. The vector to the planetary gearbox \( G_9 \) is called \( w \). The last vector, from the origin to the end effector is called \( p \). The nomenclature used to name the length of each link is the lowercase link’s name (\( a_1, b_2, c_2 \)). This same nomenclature define the vector along each link when it is in bold (\( a_1, b_2, c_2, \text{ee} \)).

III. Direct Kinematics

The direct kinematic analysis can start with the kinematic loop BB-A1-C2-B2-A2-BB. This is a five-bar linkage mechanism with the passive revolution joint \( R_7 \) as the end-effector. This kind of mechanism has been studied in many different papers [5, 6]. The position of the passive joint \( R_7 \) can be calculated by the intersection of two circles of radius \( c_2 \) and \( b_2 \), for which two possible solutions exist. The centers of these circles are at the joints \( R_4 \) and \( R_5 \). The positions of these joints are known as

\[
u_i = o_i + a_i, \quad i = 1, 2, 3 \quad (1)
\]

where

\[
a_i = \begin{bmatrix} a_i \cos \theta_i \\ a_i \sin \theta_i \\ 0 \end{bmatrix}, \quad i = 1, 2, 3 \quad (2)
\]

The position of joint 5 relative to 4 is computed as

\[
t_i = u_i - u_p, \quad i = 2, 3 \quad (3)
\]

\[
q_i = \frac{(||i||^2 - ||b||^2)}{2||v||^2}, \quad i = 2, 3 \quad (4)
\]

The two possible positions of the joint \( R_7 \) (\( v_2 \)) is calculated as

\[
v_i = u_1 + q_i t_i \pm \left( \frac{(||v||^2 - q_i^2)}{||v||^2} \right) \mathbf{E} t_i, \quad i = 2, 3 \quad (5)
\]

with

\[
\mathbf{E} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)
\]

Using the loop BB-A1-C3-B3-A3-BB, the same equations are used to find the two possible positions of the joint \( R_8 \) (\( v_3 \)).

To obtain the vector \( w \), the same equations can be used, since the position of the passive joints \( R_4 \) and \( R_5 \) are known as the center of the two circles formed by the links C4 and C5.

In this particular case, there are two possible positions for each combination of \( v_2 \) and \( v_3 \). These different positions are known as assembly configurations. As shown in Fig. 4, this manipulator architecture has eight assembly configurations. However, four of them seem to be always in a singular position (Fig 4, e to h), where no planar movement is possible. This singularity occurs because the position of the joint \( G_9 \) is always at the same position as the passive joint \( R_4 \). These configurations are not of interest and are discarded because the assumption of a parallelogram is no longer valid (links C2 and C4 are not parallel anymore and the same happens for links C3 and C5).
To know the exact position of the end effector, the orientation of this one must be known and will be called \( \theta_{ee} \). As described in the introduction and since there are only two links attached to the end effector, the proposed solution to control the orientation is a planetary gearbox. This kind of gearbox can be designed for different ratio \( r \). The orientation of the end effector is then defined by the orientation of the links \( C_4 \) and \( C_5 \). These orientations are called \( \beta_i \) and are calculated as

\[
\beta_i = \cos^{-1} \left( \frac{c_{ix}}{\|c_i\|} \right)
\]  
(7)

The orientation can then be calculated as

\[
\theta_{ee} = \beta_4 + r(\beta_4 - \beta_5)
\]  
(8)

The final vector \( ee \) can be calculated as

\[
ee = \begin{bmatrix}
    ee \cos \theta_{ee} \\
    ee \sin \theta_{ee} \\
    0
\end{bmatrix}
\]  
(9)

The final coordinate are calculated as

\[
p = w + ee
\]  
(10)

**IV. Jacobian Matrix**

One of the most significant methods to evaluate parallel manipulator properties is the Jacobian matrix. As described in [7], parallel manipulators have two of such matrices, known as the parallel Jacobian \( J_p \) and the serial Jacobian \( J_s \). The end effector velocity equation in terms of the actuated joint velocities is

\[
J_p \dot{x} = J_s \dot{\theta}
\]  
(11)

where

\[
\dot{x} = \begin{bmatrix}
    \dot{p}_x \\
    \dot{p}_y \\
    \dot{\theta}_{ee}
\end{bmatrix}, \quad \dot{\theta} = \begin{bmatrix}
    \dot{\theta}_1 \\
    \dot{\theta}_2 \\
    \dot{\theta}_3
\end{bmatrix}
\]  
(12)

These Jacobian matrices are calculated with the geometrical method described in [7]. First of all, three loop-closure equations must be defined for this mechanism. The first loop should involves \( R_1 \), which can be BB-A1-C2-C5-EE-BB. This first loop-closure equation is defined as

\[
p - ee = a_4 + c_5 + c_6
\]  
(13)

Let \( k \) be a unit vector along the positive \( z \) axis. The time derivative of the Eq. (13) becomes

\[
\dot{p} - (k \times ee) \dot{\theta}_{ee} = (k \times a_4) \dot{\theta}_1 + (k \times c_2) \dot{\theta}_2 + (k \times c_5) \dot{\theta}_5
\]  
(14)

Similarly, the time derivative of Eq. (8) gives the following

\[
\dot{\beta}_5 = \frac{\dot{\theta}_5 (1 + r) - \dot{\theta}_{ee}}{r}
\]  
(15)

Furthermore, if the four-bar mechanism forms a parallelogram, as described in section II, the following equation is true

\[
\dot{\beta}_2 = \dot{\beta}_4
\]  
(16)

Substituting Eqs. (15) and (16) in Eq. (14) yields

\[
\dot{p} + \left( k \times \left( \frac{1}{r} \right) c_5 - ee \right) \dot{\theta}_{ee} =
\]

\[
(k \times a_4) \dot{\theta}_1 + \left( k \times \left( c_2 + \frac{1 + r}{r} c_5 \right) \right) \dot{\theta}_4
\]  
(17)
The term $\dot{\beta}_4$ represents a passive revolute joint and should be eliminated. To do so, both side of Eq. (17) is dot multiplied by the $\dot{\beta}_4$ term, which finally leads to

$$\left(c_2 + \left(\frac{1+\tau}{\tau}\right)c_3\right)\dot{p} + k^T\left(\left(\frac{1}{\tau}\right)c_4 - ee\right)\times \left(c_2 + \left(\frac{1+\tau}{\tau}\right)c_3\right)\dot{\theta}_{ee} = k^T\left(a_1 \times \left(c_2 + \left(\frac{1+\tau}{\tau}\right)c_3\right)\right)d_1$$

(18)

The second loop (BB-A2-B2-C5-EE-BB) equation is developed similarly

$$p - ee = a_2 + b_2 + c_5$$

(19)

$$\dot{p} - (k \times ee)\dot{\theta}_{ee} = (k \times a_2)\dot{\theta}_2 + (k \times b_2)\dot{\theta}_2 + (k \times c_3)\dot{\theta}_3$$

(20)

where $\dot{\theta}_3 = \dot{\beta}_3 = 0$ since $\dot{a}_1 = \dot{u}_3 = 0$. To eliminate the last passive joint $\dot{\theta}_2$, which is the orientation of link B2, both side of Eq. (20) are dot multiplied by $b_2$ and the final equation for this loop becomes

$$b_2^T \dot{p} - k^T (ee \times b_2)\dot{\theta}_{ee} = k^T (a_2 \times b_2)\dot{\theta}_2$$

(21)

The third loop (A3-B3-C4-EE) equation is similar to the second and is

$$b_3^T \dot{p} - k^T (ee \times b_3)\dot{\theta}_{ee} = k^T (a_3 \times b_3)\dot{\theta}_3$$

(22)

Both Jacobian matrices are developed from Eqs. (18), (21) and (22)

$$J_p = \begin{bmatrix} j_{p11} & j_{p12} & j_{p13} \\ b_{x1} & b_{y2} & ee_x b_{2y} - ee_y b_{2x} \\ b_{z3} & b_{y4} & ee_x b_{3y} - ee_y b_{3x} \end{bmatrix}$$

(23)

$$J_s = \begin{bmatrix} j_{s31} & 0 & 0 \\ 0 & a_{2x} b_{2y} - a_{2y} b_{2x} & 0 \\ 0 & 0 & a_{3x} b_{3y} - a_{3y} b_{3x} \end{bmatrix}$$

(24)

where

$$j_{p11} = \left(c_{2x} + \left(\frac{1+\tau}{\tau}\right)c_{sx}\right)$$

(25)

$$j_{p12} = \left(c_{2y} + \left(\frac{1+\tau}{\tau}\right)c_{sy}\right)$$

(26)

$$j_{p13} = \left(\left(\frac{1}{\tau}\right)c_{sx} - ee_x\right)\left(c_{2y} + \left(\frac{1+\tau}{\tau}\right)c_{sy}\right)$$

(27)

and

$$j_{s31} = a_{1x} \left(c_{2y} + \left(\frac{1+\tau}{\tau}\right)c_{sy}\right) - a_{1y} \left(c_{2x} + \left(\frac{1+\tau}{\tau}\right)c_{sx}\right)$$

(28)

V. Singularities

Parallel manipulators can pass through two kinds of singularities if either one of the Jacobian matrices has its determinant equal to zero.

The serial singularities occur when the determinant of the matrix $J_s$ is null and this happens if one of his diagonal terms equals zero. These singularities signify that infinitesimal changes in the actuated joints have no effect on the end-effector. For example, this happen if links A1, C2 and C5 are all aligned and/or if links A2 and B2 are aligned and/or if links A3 and B3 are aligned. Other situations can eliminate the $J_{s31}$ term.

The parallel singularities, when the determinant of Jacobian $J_p$ is equal to zero, signify that infinitesimal movement of the end-effector can occur when all the actuated joints are locked. These singularities are harder to identify.

VI. Workspace

For the further analysis, the length of all links will be chosen arbitrarily as 6cm and the ratio $\tau$ as 10.

For the workspace analysis, only the position of the planetary gearbox, joint $G_p$, and the orientation of the end-effector is considered.

The position workspace of this manipulator is divided in different assembly modes. The four workspaces presented in Figs. 5-8 are the possible x and y positions of the end-effector for each assembly mode. It can be observed that the paired configurations #1, #4 and #2, #3 are symmetrical. This symmetrical property is only due to the position of the actives joints, which they are positioned along the X-axis. The entire workspace, Fig 9, can be obtained theoretically by the intersections of the area covered by the combination of links (A1, C2 and C5), (A2, B2 and C5) and (A3, B3 and C4) as per Fig. 10, where the available workspace is the intersection of the blue, green and red circle.
Figure 5 – Position workspace for assembly configuration #1

Figure 6 – Position workspace for assembly configuration #2

Figure 7 – Position workspace for assembly configuration #3

Figure 8 – Position workspace for assembly configuration #4

Figure 9 – Complete position workspace

Figure 10 – Theoretical Workspace Analysis
The integration of the end-effector orientation in the workspace analysis is more complex, because multiple solutions can lead to the same orientation, due to the planetary gearbox.

With the constant orientation workspace method, described in [8], it is possible to find the available workspace for a specific configuration and orientation of the articulated platform. For a specific configuration and orientation of the platform, the platform is considered as solid, movable only in x and y-axis. For the specific case where the parallelogram forms a square and the diagonal from \( R_4 \) to \( G_9 \) is along y-axis, the available workspace, in Fig. 11, is the intersection of the red segment with the interior of the blue and green circle. In this particular case, all positions on the red circle are accessible. The reason why only the red circle is included instead of all the area inside it is because the first limb does not have a passive link between the first link and the articulated platform. For this manipulator, this method does not consider that multiple configuration of the articulated platform can result in the same orientation. This means that an infinite number of analyses can be performed to obtain the entire workspace of the manipulator for a given orientation.

Figure 11 - Example of position accessible for a constant orientation

An empirical method can give an idea of the orientation possibilities of this manipulator. The Fig. 12 and 13 show two different randomly chosen orientation workspaces. These workspaces appear to be similar and cover almost the entire available workspace, which demonstrates that almost all positions are achievable for any orientation. To be noted that these results do not guarantee that a constant orientation trajectory is achievable on the entire workspace. However, as seen previously, there are constant orientation trajectories possible for each position.

Figure 12 - Workspace for orientation 0 to 2\pi/100 rad

Figure 13 - Workspace for 30^\circ 2\pi/100 to 31^\circ 2\pi/100 rad
V. Conclusions

This paper presents a new architecture for a planar parallel manipulator, which has an interesting workspace due to multiple orientation configurations achievable by the planetary gearbox. This gearbox could have different ratios for different applications, which can result in a different available workspace. However, this modification can affect the dexterity of the manipulator.

Further work can be done to know the effect of any change of the links’ length on the workspace or the effect of the positions of the active joints $R_1$, $R_2$ and $R_3$. A prototype can be built.

Even further work can also be done to transpose this architecture to spherical motion.

References